JP Maths Revision

ITERATION

HIGHER TIER

Attempt the paper before watching the solutions!

https://www.youtube.com/@JPMathsRevision

Name: ______________________ Date: ______________________

1. (a) Show that

$x^2 + x - 7 = 0$

has a solution between $x = 1$ and $x = 2$. **(2)**

At $x = 1$, $1^2 + 1 - 7 = -5 < 0$
At $x = 2$, $2^2 + 2 - 7 = 1 > 0$. Since there is a change of sign, there is a solution between $x = 1$ and $x = 2$.

(b) Show that the equation can be rearranged to give

$x = \sqrt[3]{7 - x}$ **(1)**

(c) Starting with $x_0 = 1.8$, use

$x_{n+1} = \sqrt[3]{7 - x_n}$

twice to estimate the solution.
Give your answer to 4 decimal places.

$x_1 = 1.804396$
$x_2 = 1.804675$
Estimated solution ≈ 1.8047 **(3)**

2. (a) Show that

$x^3 + 5x - 12 = 0$

has a solution between $x = 1$ and $x = 2$. **(2)**

At $x = 1$, $1^3 + 5(1) - 12 = -6 < 0$
At $x = 2$, $2^3 + 5(2) - 12 = 6 > 0$. Since there is a change of sign, there is a solution between $x = 1$ and $x = 2$.

(b) Show that the equation can be rearranged to give

$x = \dfrac{12}{x^2 + 5}$ **(1)**

(c) Starting with $x_0 = 1.6$, use

$x_{n+1} = \dfrac{12}{x_n^2 + 5}$

three times to estimate the solution.
Give your answer to 4 decimal places.

$x_1 = 1.714286$ $x_3 = 1.692144$
$x_3 = 1.689934$
Estimated solution ≈ 1.6921 **(3)**

3. (a) Show that

$x^3 + x - 4 = 0$

has a solution between $x = 1$ and $x = 2$. **(2)**

At $x = 1$, $1^3 + 1 - 4 = -2 < 0$
At $x = 2$, $2^3 + 2 - 4 = 6 > 0$. Since there is a change of sign, there is a solution between $x = 1$ and $x = 2$.

(b) Show that the equation can be rearranged to give

$x = \sqrt[3]{4 - x}$ (1)

rting with $x_0 = 1.4$, use

$x_{n+1} = \sqrt[3]{4 - x_n}$

three times to estimate the solutions.

4. A water tank contains 3500 litres of water at time $t = 0$ hours.
At the end of each hour, 4% of the water remaining leaks away.
The volume of water after t hours is V_t.
Given that $V_0 = 3500$
and
$V_{t+1} = kV_t$

(a) Write down the value of k

$k = 0.96$ **(1)**

(b) Find the volume after 5 hours.

2923.2 litres **(3)**

5. The number of fish in a lake at the start of year n is P_n.
At the start of year 1 there are 1200 fish.
Given that $P_{n+1} = 1.06P_n$

(a) Write down the percentage that the population of fish increases by each year.

6 % **(1)**

(b) Work out the number of fish at the start of year 6.

1604 **(3)**

6. The number of rabbits on a farm at the end of month n is R_n.
The number of rabbits at the end of the next month is given by $R_{n+1} = 1.15R_n - 40$.
At the end of April there are 240 rabbits.
Work out how many rabbits there will be at the end of July.

465 rabbits **(4)**

7. The value of a bicycle at the end of year n is V_n.
The value is modelled by $V_{n+1} = 0.88V_n + 30$.
The bicycle is worth £600 initially.
Work out its value after 3 years.

£490.18 **(3)**

8. A balloon is descending.
The height of the balloon n minutes after it starts to descend is h_n metres.
The height is modelled by $h_{n+1} = Kh_n + 15$, where K is a constant.
The balloon starts descending from 1500m.
After one minute its height is 1275m.

(a) Find the value of K.

$K = 0.84$ **(3)**

(b) Work out the height of the balloon after 4 minutes.

939.7 m **(3)**

9. A cup of coffee cools according to the formula
$T_{n+1} = 0.9T_n + 2$, where T_n is the temperature in degrees Celsius after n minutes.
Initially the coffee has a temperature of 92°C.
Work out the temperature after 5 minutes.
Give your answer to 1 decimal place.

54.6 °C **(3)**

10. The annual profit of a company in year n is £ P_n.
The profits follow the model $P_{n+1} = 1.08P_n + 1200$.
In 2026, the profit is £45000.
Work out the predicted profit in 2029.

£56,554.40 **(3)**

11. The value of a car is modelled by
$V_{n+1} = 0.82V_n + 450$,
where V_n is the value of the car after n years.
Initially the car is worth £18,000.

(a) Work out the value after 4 years.

£14,581.19 **(3)**

(b) Explain what the 450 represents in this model.

It represents the value that the car keeps or decreases towards – the fixed amount added each year which is the car's estimated value when it is very old. **(1)**