JP Maths Revision

ALGEBRAIC PROOF

HIGHER TIER

Attempt the paper before watching the solutions!

https://www.youtube.com/@JPMathsRevision

Name: ______________________ Date: ______________

1. Jane says that the product of any two prime numbers is always odd.
Jane is wrong. Explain why.

The product of two prime numbers is not always odd.

For example, $2 \times 2 = 4$, which is even.

Therefore, Jane is wrong.

(1 mark)

2. Jackson thinks that $n^2 + 5n + 1$ is always a prime number for every integer n.
Is he correct?

Yes ☐ No ☑

Give a reason to support your answer.

When $n = 5$, $n^2 + 5n + 1 = 25 + 25 + 1 = 51 = 3 \times 17$,

which is not prime. Therefore, Jackson is not correct.

(2 marks)

3. n is a positive integer. Aria says that expression $n^2 + 3n + 2$ can never be prime.
Explain why Aria is correct.

$n^2 + 3n + 2 = (n + 1)(n + 2)$.

Since n is a positive integer, both $n + 1$ and $n + 2$ are integers

greater than 1 and are consecutive, so their product cannot be prime.

(2 marks)

4. Susan says that the product of a square number and a cube number is always even.
Give an example to prove that Susan is wrong.

$1^2 \times 1^3 = 1 \times 1 = 1$, which is odd.

(1 mark)

5. A is a positive even integer. B is a negative integer. Complete the following table.

	Sometimes true	Always true	Never true
AB is odd	✓	–	–
$A + B$ is positive.	✓	–	–
A^2B is negative.	–	✓	–

(2 marks)

6. Match each description to the correct expression.

Any two numbers	n^2, m^2
Two consecutive numbers	$2n + 1$, $2n + 3$
Two consecutive even numbers	n, $n + 1$
Two consecutive odd numbers	n, m
Any two even numbers	$2n$, $2m$
Any two odd numbers	$8n$, $8m$
Two square numbers	$2n$, $2n + 2$
Two multiples of 8	$2n + 1$, $2m + 1$

(3 marks)

7. Prove algebraically that the sum of any odd integer and any even integer is always odd.

Let the odd integer be $2n + 1$ and the even integer be $2m$.

$(2n + 1) + 2m = 2n + 2m + 1 = 2(n + m) + 1$, which is odd.

(2 marks)

8. Prove algebraically that the sum of any two consecutive even integers is two more than a multiple of four.

Let the consecutive even integers be $2n$ and $2n + 2$.

$2n + 2n + 2 = 4n + 2 = 4n + 2$, which is two more than a multiple of four.

(2 marks)

9. Prove algebraically that the product of any two consecutive even integers is always a multiple of four.

Let the consecutive even integers be $2n$ and $2n + 2$.

$(2n)(2n + 2) = 4n(n + 1)$, which is a multiple of four.

(2 marks)

10. Prove that the product of any two consecutive odd integers is three more than a multiple of four.

Let the consecutive odd integers be $2n + 1$ and $2n + 3$.

$(2n + 1)(2n + 3) = 4n^2 + 8n + 3 = 4n(n + 2) + 3$,

which is three more than a multiple of four.

(2 marks)

11. Prove that the sum of any two even integers is always even.

Let the even integers be $2n$ and $2m$.

$2n + 2m = 2(n + m)$, which is even.

(2 marks)

12. Prove that the sum of any two consecutive numbers is odd.

Let the consecutive numbers be n and $n + 1$.

$n + (n + 1) = 2n + 1$, which is odd.

(2 marks)

13. Prove that the sum of the squares of any two consecutive odd numbers is two more than a multiple of eight.

Let the consecutive odd numbers be $2n + 1$ and $2n + 3$.

$(2n + 1)^2 + (2n + 3)^2 = (4n^2 + 4n + 1) + (4n^2 + 12n + 9)$

$= 8n^2 + 16n + 10 = 8n(n + 2) + 2$,

which is two more than a multiple of eight.

(3 marks)