Completing the Square

Exam Style Questions

1. Complete the square of $x^2 + 8x + 4$

$x^2 + 8x + 4 = (x+4)^2 - 4^2 + 4$

$= (x+4)^2 - 16 + 4$

$= (x+4)^2 - 12$

$(x+4)^2 - 12$ (2 marks)

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2. Complete the square of $x^2 + 12x + 6$

$x^2 + 12x + 6 = (x+6)^2 - 6^2 + 6$

$= (x+6)^2 - 36 + 6$

$= (x+6)^2 - 30$

$(x+6)^2 - 30$ (2 marks)

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3. Complete the square of $x^2 + 20x - 4$

$= (x+10)^2 - 10^2 - 4$

$= (x+10)^2 - 100 - 4$

$= (x+10)^2 - 104$

$(x+10)^2 - 104$ (2 marks)

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4. (a) Write $x^2 + 12x + 6$ in the form $(x + a)^2 + b$ where a and b are integers.

$= (x+6)^2 - 6^2 + 6$

$= (x+6)^2 - 36 + 6$

$= (x+6)^2 - 30$

$(x+6)^2 - 30$

(b) Hence, write down the coordinates of the turning point of the graph with equation $y = x^2 + 12x + 6$

$(-6, -30)$ (3 marks)

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5. (a) Write $x^2 + 9x - 1$ in the form $(x + a)^2 + b$ where a and b are constants to be determined.

$= (x+4.5)^2 - 4.5^2 - 1$

$= (x+4.5)^2 - 20.25 - 1$

$= (x+4.5)^2 - 21.25$

$(x+4.5)^2 - 21.25$

(b) Hence, write down the coordinates of the turning point of the graph with equation $y = x^2 + 9x - 1$

$(-4.5, -21.25)$ (3 marks)

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6. (a) Write $x^2 - 20x + 3$ in the form $(x + a)^2 + b$ where a and b are constants to be determined.

$= (x-10)^2 - 10^2 + 3$

$= (x-10)^2 - 100 + 3$

$= (x-10)^2 - 97$

$(x-10)^2 - 97$

(b) Hence, write down the coordinates of the turning point of the graph with equation $y = x^2 - 20x + 3$

$(10, -97)$ (3 marks)

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7. By completing the square, find the coordinates of the turning point of the graph with equation $y = x^2 - x - 5$.

$y = \left(x - \frac{1}{2}\right)^2 - \left(\frac{-1}{2}\right)^2 - 5$

$= \left(x - \frac{1}{2}\right)^2 - \frac{1}{4} - 5$

$= \left(x - \frac{1}{2}\right)^2 - \frac{21}{4} \Rightarrow TP = \left(\frac{1}{2}, -\frac{21}{4}\right)$

$= (0.5, -5.25)$

$(0.5, -5.25)$ (3 marks)

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8. By completing the square, find the coordinates of the turning point of the graph with equation $y = 5x^2 + 10x + 20$.

$y = 5[x^2 + 2x + 4]$

$= 5[(x+1)^2 - 1^2 + 4]$

$= 5[(x+1)^2 + 3]$

$= 5(x+1)^2 + 15 \Rightarrow TP = (-1, 15)$

(-1, 15) ..... (4 marks)

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9. By completing the square, find the coordinates of the turning point of the graph with equation $y = 3x^2 + 12x + 18$.

$y = 3[x^2 + 4x + 6]$

$= 3[(x+2)^2 - 2^2 + 6]$

$= 3[(x+2)^2 - 4 + 6]$

$= 3[(x+2)^2 + 2]$

$= 3(x+2)^2 + 6$

(-2, 6) ..... (4 marks)

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10. By completing the square, find the coordinates of the turning point of the graph with equation $y = 2x^2 + 8x + 9$.

$y = 2[x^2 + 4x + 4.5]$

$= 2[(x+2)^2 - 2^2 + 4.5]$

$= 2[(x+2)^2 - 4 + 4.5]$

$= 2[(x+2)^2 + 0.5]$

$= 2(x+2)^2 + 1$

(-2, 1) ..... (4 marks)

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11. By completing the square, find the coordinates of the turning point of the graph with equation $y = 3x^2 + 7x - 3$.

$y = 3\left[x^2 + \frac{7}{3}x - 1\right]$

$= 3\left[\left(x + \frac{7}{6}\right)^2 - \left(\frac{7}{6}\right)^2 - 1\right]$

$= 3\left[\left(x + \frac{7}{6}\right)^2 - \frac{49}{36} - 1\right]$

$= 3\left[\left(x + \frac{7}{6}\right)^2 - \frac{85}{36}\right] = 3\left(x + \frac{7}{6}\right)^2 - \frac{85}{12}$

$\left(-\frac{7}{6}, -\frac{85}{12}\right)$ ..... (4 marks)

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12. By completing the square, solve the equation $x^2 + 6x + 2 = 0$ leaving your solutions in the form $a \pm b\sqrt{c}$ where a, b and c are integers.

$(x+3)^2 - 3^2 + 2 = 0$

$(x+3)^2 - 9 + 2 = 0$

$(x+3)^2 - 7 = 0$

$(x+3)^2 = 7$

$x + 3 = \pm\sqrt{7}$

$x = -3 \pm \sqrt{7}$

$x = -3 \pm \sqrt{7}$ ..... (4 marks)

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13. By completing the square, solve the equation $x^2 + 7x + 11 = 0$ leaving your solutions in surd form.

$(x + 3.5)^2 - 3.5^2 + 11 = 0$

$(x+3.5)^2 - 12.25 + 11 = 0$

$(x+3.5)^2 - 1.25 = 0$

$(x+3.5)^2 = 1.25$

$x + 3.5 = \pm\sqrt{1.25}$

$x = -3.5 \pm \sqrt{1.25}$

$= -\frac{7}{2} \pm \sqrt{\frac{5}{4}}$

$= \frac{-7 \pm \sqrt{5}}{2}$

$x = \frac{-7+\sqrt{5}}{2}$ or $x = \frac{-7-\sqrt{5}}{2}$

..... (4 marks)

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14. By completing the square, solve the equation $2x^2 + 8x - 2 = 0$ leaving your solutions in the form $a \pm b\sqrt{c}$ where a, b and c are integers.

$2[x^2+4x-1] = 0$

$x^2+4x-1=0$

$(x+2)^2-2^2-1=0$

$(x+2)^2-5=0$

$(x+2)^2=5$

$x+2 = \pm\sqrt{5}$

$x = -2\pm\sqrt{5}$

$x = -2 \pm \sqrt{5}$ ..... (5 marks)

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15. By completing the square, solve the equation $-x^2 - 7x + 2 = 0$ leaving your solutions in surd form.

$-x^2-7x+2=0$

$\Rightarrow x^2+7x-2=0$

$\Rightarrow (x+3.5)^2-3.5^2-2=0$

$\Rightarrow (x+3.5)^2-12.25-2=0$

$(x+3.5)^2-14.25=0$

$(x+3.5)^2=14.25$

$x+3.5 = \pm\sqrt{14.25}$

$x = -3.5 \pm \sqrt{14.25}$

$= -\frac{7}{2} \pm \sqrt{\frac{57}{4}}$

$= \frac{-7\pm\sqrt{57}}{2}$

$x = \frac{-7\pm\sqrt{57}}{2}$ ..... (5 marks)

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16. By completing the square, solve the equation $ax^2 + bx + c = 0$, leaving your answer in terms of a, b and c.

$$a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] = 0$$

$$\Rightarrow \quad x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$\Rightarrow \quad \left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{4ac}{4a^2}$$

$$\Rightarrow \quad x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\Rightarrow \quad x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

..... (6 marks)